**Three Phase Load Flow for Unbalanced Power Systems**

Sugiarto 1), Hadi, S.P 2), Tumiran 2)

1) Electrical Engineering Department, Sekolah Tinggi Teknologi Nasional, Yogyakarta

sugiarto.kadiman@gmail.com

**2**) Electrical Engineering Department, Gadjah Mada University, Yogyakarta

sasongko@te.ugm.ac.id tumiran@te.ugm.ac.id

**Abstract** – One of the problems encountered in power system is the appearance of unbalanced voltages and currents in the presence of long untransposed transmission lines or inconcistanced loading formed by users. To asses these unbalance effects in any detail, a 3-phase-load-flow solution that allows to represent of all possible unbalances as they occur in the power systems without making any assumptions is essential. Consequently, a working three phase load flow in the phase frame of reference which based on the sequence component frame and power balance is presented, which forms the basis of a computer program developed for the specific purpose of solving the 3-phase-load-flow problem such as the inductive and resistive effects of multiconductor transmission lines and transmission line losses. To show an assessment of the proposed three phase load flow into unbalance problem, 5-bus transmission networks without transpositions were tested. Moreover, the results reveal that solving the load flow problems with Newton-Raphson can considerably work by simulation program when comparing with some scenarios of unbalanced load of the test network.

 **Keywords**: unbalanced voltages and currents, untransposed, sequence component frame

### 1. INTRODUCTION

 As the number of extra-high-voltage transmission lines increases, particularly in areas where huge power stations large and remote load centers through lines with no or few transpositions, the unbalance effects of these transmission lines with any possible unbalance in load or source must be properly analyzed. The unbalance effect may not be especially significant as far as the network itself is concerned, but in terms of the individual components of the network it can be extremely important, such as the presence of negative-sequence current at the generator terminals which will give rise to heating in its rotor and increase power loss.

 Theoretically, a three-phase power system is a complex alternating circuit. However, unknown three-phase voltage magnitudes and angles on the buses in three-phase load flow problems have to be calculated for a given values of three-phase active and reactive power injections or specified conditions on voltages. Fortunately, a three-phase load flow can be solved in a similar way to single-phase load flow except for extending the dimensions of the system admittance or impedance matrix and variable vector, if an internal bus is introduced to every generator and is treated as an independent bus.

 Unbalance problem motivated the development of a three phase load flow algorithm for assessment of power system unbalance. Kalyuzhny and Kusnir (2007) shows short lines feeding balanced loads may have circulation current asymetry on their transmission lines. Emin and Crisford (2006) discusses a negative phase sequence voltage issue on the transmission system. Relatively old method using symmetrical component theory or phase and sequence components have been applied to asses power quality problems in the transmission system (Abdel-Akher *et al*, 2005; Smith and Arrilaga, 1998; Zang and Chen, 1994). This paper aims at developing an algorithm which can cope with model of the inductive and resistive effects of multiconductor transmission lines as a series

impedance matrix, and the capacitive effects as a shunt admittance matrix. The choice of method for any load-flow solution depends on many factors, the most important being convergence characteristics, reliability and computation load (Chen *et al*, 1990). Although the classical Newton-Raphson method is very efficient and becomes the standard for most the power flow calculation, to formulate a matrix equation requires tedious and complicated mathematical expressions (Chen and Yang, 2009). In this paper, the Newton-Raphson method is still applied. The mismatch to formulate the matrix equation is derived directly from the power-balanced equations. For illustration, a sample 5-bus network is also presented.

**2. TRANSMISSION SYSTEM MODEL**

 In three-phase systems, a balanced transmission line is typically represented only by its positive-sequence series impedance in addition to two shunt admittances ($π$ model). The equivalent circuit representation of a three phase transmission line section is shown in Figure 1 (Peralta *et al*, 2008). The effect of mutual coupling and ground have been included in this equivalent diagram. A driving point admittance matrix $Y\_{abc}$ which is derived by multiplying the inverse of primitive impedance matrix $Z\_{abc}$ by its branch-bus incidence matrix can be shown by the expression 1.

$Y\_{abc}=N^{T}Z^{-1}N=\left[\begin{array}{c}Y\_{kk}^{abc} -Y\_{km}^{abc}\\-Y\_{mk}^{abc} Y\_{mm}^{abc}\end{array}\right]$ (1)

where

$Y\_{kk,mm,km,mk}^{abc}=\left[\begin{array}{c}y\_{aa} y\_{ab} y\_{ac}\\y\_{ba} y\_{bb} y\_{bc}\\y\_{ca} y\_{cb } y\_{cc}\end{array}\right]$ (2)

The mutual capacitive element represented by matrix $B\_{abc}$ is determined as follows:

$B\_{abc}=\left[\begin{array}{c}B\_{kk}^{abc} 0 \\ 0 B\_{mm}^{abc}\end{array}\right]$ (3)

where

$B\_{kk,mm}^{abc}=\left[\begin{array}{c}b\_{aa} b\_{ab} b\_{ac}\\b\_{ba} b\_{bb} b\_{bc}\\b\_{ca} b\_{cb} b\_{cc}\end{array}\right]$ (4)

Therefore six additional series and six shunt capacitive element need to be entered in the system admitance matrix. The susceptance $-b\_{ab}$, $-b\_{bc}$, and $-b\_{ac}$ in Figure 1 should not add up to the sum diagonal element of the system matrix. The final matrix for the transmission line section has following form:

$Y\_{abc}=\left[\begin{array}{c}Y\_{kk}^{abc}+B\_{kk}^{abc} -Y\_{km}^{abc} \\-Y\_{mk}^{abc} Y\_{mm}^{abc}+B\_{mm}^{abc}\end{array}\right]$ (5)

 It is necessary for the unbalanced load flow calculation to do modeling accurately. The node equation is shown in the power system for the load flow calculation, and the three phase models are built into this.

$$y\_{aa}$$

$$k\_{a}$$

$$k\_{c}$$

$$k\_{b}$$

$$-y\_{ac}$$

$$-y\_{ac}$$

$$-y\_{ab}$$

$$-y\_{bc}$$

$$-y\_{ab}$$

$$-y\_{bc}$$

$$y\_{ac}$$

$$y\_{ba}$$

$$y\_{ab}$$

$$y\_{cb}$$

$$y\_{cc}$$

$$y\_{aa}$$

$$y\_{bc}$$

$$y\_{bb}$$

$$-b\_{ab}$$

$$-b\_{bc}$$

$$b\_{aa}$$

$$b\_{bb}$$

$$b\_{cc}$$

$$-b\_{ac}$$

$$-b\_{ab}$$

$$-b\_{bc}$$

$$b\_{aa}$$

$$b\_{bb}$$

$$b\_{cc}$$

$$-b\_{ac}$$

$$m\_{a}$$

$$m\_{a}$$

$$m\_{a}$$

Figure 1. Positive-sequence equivalent circuit of a three-phase line section

Moreover, the formulations are expressed by *a*, *b*, and *c* phase sequence component to keep easily. It only has to convert Equation 1 by using Equation 6 and 7 for the symmetrical coordinates form.

$\left.\begin{array}{c}∆V\_{012}=T\_{S}∆V\_{abc}\\I\_{012}=T\_{S}^{-1}∆V\_{abc}\\Z\_{012}=T\_{S}^{-1}Z\_{abc}T\_{S}\end{array} \right\}$ (6)

$\left.\begin{array}{c}T\_{S}=\left[\begin{array}{c}1 1 1\\1 h^{2} h\\ 1 h h^{2}\end{array}\right]\\h=1∠120^{o} and h^{2}=1∠240^{o}\end{array} \right\}$ (7)

**3. PROBLEM FORMULATION**

 To analyze system characteristics, nonlinear nodal analysis is employed to formulate a set of complex load flow equations as shown in Equation 4. Also, these equation can be decomposed into real and reactive power equations as in Equation 5 and 6, where, $P\_{k}^{cal}$ and $Q\_{k}^{cal}$ are calculated real power and calculated reactive power, respectively. The proposed three-phase load flow based on positive sequence nodal power simplifies these equations by rearranging into Equation 11– 13

 $S\_{k}=P\_{k}+jQ\_{k}=E\_{k}I\_{k}^{\*}=E\_{k}\left(Y\_{kk}E\_{k}+Y\_{km}E\_{m}\right)^{\*}$ (8)

 $P\_{k}^{cal}=V\_{k}^{2}G\_{kk}+V\_{k}V\_{m}\left[G\_{km}\cos(\left(θ\_{k}-θ\_{m}\right)+B\_{km}\sin(\left(θ\_{k}-θ\_{m}\right)))\right]$ (9)

 $Q\_{k}^{cal}=-V\_{k}^{2}B\_{kk}+V\_{k}V\_{m}\left[G\_{km}\sin(\left(θ\_{k}-θ\_{m}\right)-B\_{km}\cos(\left(θ\_{k}-θ\_{m}\right)))\right]$ (10)

 $\left[\begin{array}{c}S\_{k}^{abc}\\S\_{m}^{abc}\end{array}\right]=\left[\begin{array}{c} P\_{k}^{abc}+jQ\_{k}^{abc}\\ P\_{m}^{abc}+jQ\_{m}^{abc} \end{array}\right]\left[\begin{array}{c}E\_{k}^{abc}I\_{k}^{abc\*}\\E\_{m}^{abc}I\_{m}^{abc+}\end{array} \right]$ (11)

 $P\_{k}^{ρ}=V\_{k}^{ρ}\left.\left\{\sum\_{i=k,m}^{}\sum\_{j=a,b,c}^{}V\_{l}^{j}\right.\left[G\_{ki}^{ρj}cos\left(θ\_{k}^{ρ}-θ\_{i}^{j}\right)+B\_{ki}^{ρj}sin\left(θ\_{k}^{ρ}-θ\_{i}^{j}\right)\right]\right\}$ (12)

 $Q\_{k}^{ρ}=V\_{k}^{ρ}\left.\left\{\sum\_{i=k,m}^{}\sum\_{j=a,b,c}^{}V\_{l}^{j}\right.\left[G\_{ki}^{ρj}sin\left(θ\_{k}^{ρ}-θ\_{i}^{j}\right)-B\_{ki}^{ρj}cos\left(θ\_{k}^{ρ}-θ\_{i}^{j}\right)\right]\right\}$ (13)

where, $P\_{k}^{ρ}$ and $Q\_{k}^{ρ}$ are active and reactive powers injected at phases *a, b,* and c of bus *k*, respectively. Therefore, power mismatch equations are used to formulate the proposed Newton-Raphson updating as shown in Equation 15. To update node-voltage vectors, elements of the Jacobian matrix must be calculated. Jacobian sub-matrices can be expressed as in Equation 10.

 $\left[ \begin{array}{c}∆P\_{l}^{ρ}\\∆Q\_{l}^{ρ}\end{array} \right]=\left[ \begin{array}{c} \\J\_{1} J\_{2}\\J\_{3} J\_{4}\end{array}\right]\left[ \begin{array}{c}∆θ\_{j}^{ρ}\\\frac{∆V\_{j}^{ρ}}{V\_{j}^{ρ}}\end{array} \right]$ (14)

With this computation, voltage magnitudes and phases can be updated iteratively by using the following equation where *i* indicates a counter for iteration.

 $\left[\begin{array}{c}θ\_{j}^{ρ}\\ V\_{j}^{ρ} \end{array}\right]^{(i+1)}=\left[ \begin{array}{c}θ\_{j}^{ρ}\\V\_{j}^{ρ }\end{array}\right]^{(i)}+ \left[ \begin{array}{c} \\J\_{1} J\_{2}\\J\_{3} J\_{4} \end{array}\right]^{-1}\left[ \begin{array}{c}∆P\_{j}^{ρ}\\∆Q\_{j}^{ρ}\end{array} \right]^{(i)}$ (15)

In addition, a power flow solution framework can be summarized in flow diagram of Fig. 2.

START

Load system data

Initialize node volatges

Formulate *Ybus*

Calculate $∆P\_{j}^{ρ}$,$ ∆Q\_{j}^{ρ}$

Calculate

Jacobian sub-matrices

*J1, J2, J3* and *J4*

 Update nodal voltage, *Vbus*

Convergence ?

Show results

STOP

YES

NO

Figure 2. Flow diagram for load flow calculation

**4. SIMULATION RESULTS**

 The effectiveness of the proposed load flow was test against 5-bus transmission networks which is slightly modified from Pai (1979), as shown in Figure 4. The data of lines, bus voltage, generating and load capacity are shown in Table 1 -2, respectively.

Table 1. The line data of 5-bus test network

|  |  |  |
| --- | --- | --- |
| BusFr. To | **Positive Sequence** ***R* *X* *G B*** | **Zero Sequence** ***R X G B*** |
| 1 2 | 0.02 0.06 0 0.06 | 0.06 0.18 0 0.18 |
| 1 3 | 0.08 0.24 0 0.05 | 0.24 0.72 0 0.15 |
| 2 3 | 0.06 0.18 0 0.04 | 0.18 0.54 0 0.12 |
| 2 4 | 0.06 0.18 0 0.04 | 0.18 0.54 0 0.12 |
| 2 5 | 0.04 0.12 0 0.03 | 0.12 0.36 0 0.09 |
| 3 4 | 0.01 0.03 0 0.02 | 0.03 0.09 0 0.06 |
| 4 5 | 0.08 0.24 0 0.05 | 0.24 0.72 0 0.15 |

**LEGEND**

$$G2$$

**4**

**5**

$$G1$$

**1**

**2**

**3**

**2 : bus**

**:load**

$G1$ **: generator**

Figure 3. 5-Bus tested network (Pai, 1979)

Table 2. Voltage, generator and load capasity

|  |  |  |
| --- | --- | --- |
|  BusNo. Voltage | GenerationMW MVAR | LoadMW MVAR |
| 1 | 50000 |  Slack Bus |  0 0 |
| 2 | 50000 | 40.0 0.0 |  20 10 |
| 3 | 50000 | 0 0 |  45 15 |
| 4 | 50000 | 0 0 |  40 5 |
| 5 | 50000 | 0 0 |  60 10 |

 The following types of unbalance are considered:

**Unbalance type A:** At first the overall network load is balanced for three phases. Afterward, a percentage of the load of phase *b* load is decreased by 5%, while the same value is increased in phase *c* by 5%. The total network remains constant under each unbalanced scenario.

**Unbalance type B:** Firstly the overall network load is balanced for three phases. Subsequently, a percentage of the load of phase *b* load is decreased by 5%, while the decreased is twice in phase *c* or by 10%. This kind of unbalance reduces the total network under each unbalanced scenario.

 Table 3 shows results of unbalance analysis for 5-bus test network when two unbalance scenarios was applied. The total loss of transmission lines is shown in Table 4.

 It is noticed that despite demand being maintained, total losses calculated by test network increased for a 5% unbalance compared to a fully balanced system. Due to the decrease in demand under Unbalance Type B scenario, the amount of losses also decreased.

Table 3. Result of bus voltages

|  |  |
| --- | --- |
| BUSBARNo Volt. Ph. | SCENARIO |
| **Balance** | **Unbalance** **Type A** | **Unbalance** **Type B** |
| 1 | **Mag.****(Kv)** | ***A*** | 500 | 500 | 500 |
| ***B*** | 500 | 500 | 500 |
| ***C*** | 500 | 500 | 500 |
| **Phase****(deg.)**  | ***A*** | 0 | 0 | 0 |
| ***B*** | 240 | 240 | 240 |
| ***C*** | 120 | 120 | 120 |
| 2 | **Mag.** | ***A*** | 500 | 500 | 495.05 |
| ***B*** | 500 | 500 | 497.15 |
| ***C*** | 500 | 500 | 497.20 |
| **Phase****(deg.)**  | ***A*** | - 3.39 | - 3.29 | - 3.41 |
| ***B*** | 236.61 | 236.94 | 236.98 |
| ***C*** | 116.61 | 116.17 | 117.36 |
|  | **Mag.****(Kv)** | ***A*** | 488.55 | 487.30 | 495.70 |
| ***B*** | 488.55 | 488.15 | 496.90 |
| 3 | ***C*** | 488.55 | 482.25 | 496.95 |
|  | **Phase****(deg.)** | ***A******B******C*** | - 5.74234.26114.26 | - 5.82234.92113.67 | - 6.38234.33114.79 |
| 4 | **Mag.****(Kv)** | ***A******B******C*** | 485.60485.60485.60 | 487.40484.15482.20 | 500500500 |
| **Phase****(deg.)** | ***A******B******C*** | - 6.12233.88113.88 | - 6.20234.59113.25 | - 6.93233.91114.32 |
| 5 | **Mag.****(Kv)** | ***A******B******C*** | 483.65483.65483.65 | 485.70485.30479.95 | 483.35488.70488.35 |
| **Phase****(deg.)**  | ***A*** | - 7.06 | - 7.06 | - 7.44 |
|  | ***B*** | 232.94 | 233.76 | 233.23 |
| ***A*** | 112.94 | 112.20 | 114.08 |

Table 4. Transmission line losses for unbalance scenarios

|  |  |
| --- | --- |
| **BUSBAR****From To Phases** |  **SCENARIO** |
|  **BALANCE** |  **UNBALANCE TYPE A** |  **UNBALANCE TYPE B** |
| **Sending Receiving** **Active Reactive Active Reactive****(MVA) (MVAR) (MVA) ( MVAR)** |  **Sending Receiving****Active Reactive Active Reactive****(MVA) (MVAR) (MVA) (MVAR)** |  **Sending Receiving****Active Reactive Active Reactive****(MVA) (MVAR) (MVA) (MVAR)** |
| **1** | **2** | ***A******B******C*** |  89.68 - 29.97 89.68 - 29.97 89.68 - 29.97 | - 87.93 29.23- 87.93 29.23- 87.93 29.23 |  88.89 - 25.54 83.04 - 31.11 97.11 - 33.21 | - 87.54 24.60- 81.44 29.51- 94.85 33.64 |  89.93 - 17.46 82.26 - 28.70 76.29 - 18.33 | - 87.58 - 16.65 - 81.11 - 11.92 - 75.03 - 15.87  |
| **1** | **3** | ***A******B******C*** |  40.64 - 1.99 40.64 - 1.99 40.64 - 1.99 | - 39.32 1.09- 39.32 1.09- 39.32 1.09 |  40.44 - 1.21 38.16 - 2.92 43.34 - 1.78 | - 39.32 0.34- 37.88 1.33- 42.61 1.63 |  41.59 - 11.25 38.58 - 9.01 36.32 - 11.06 | - 39.99 10.76- 37.51 7.73- 35.21 9.30 |
| **2** | **3** | ***A******B******C*** |  24.82 6.22 24.82 6.22 24.82 6.22 | - 24.41 - 8.87- 24.41 - 8.87- 24.41 - 8.87 |  24.74 5.81 23.62 5.56 26.11 7.31 | - 24.42 - 8.48- 23.30 - 8.44- 25.22 - 9.71 |  24.83 - 10.21 23.58 - 8.34 22.52 - 9.26 | - 24.36 7.60- 23.25 10.07- 25.06 11.28 |
| **2** | **4** | ***A******B******C*** |  28.08 5.27 28.08 5.27 28.08 5.27 | - 27.57 - 7.64- 27.57 - 7.64- 27.57 - 7.64 |  27.99 4.94 26.67 4.61 29.59 6.30 | - 27.58 - 7.32- 26.27 - 7.25- 28.87 - 8.36 |  28.32 - 15.43 26.78 - 12.60 25.55 - 13.82 | - 27.68 13.35- 26.33 10.07 - 25.06 11.28 |
| **2** | **5** | ***A******B******C*** |  55.03 9.08 55.03 9.08 55.03 9.08 | - 53.78 - 8.22- 53.78 - 8.22- 53.78 - 8.22 |  54.81 8.94 52.15 7.92 58.15 10.45 | - 53.80 - 8.00- 51.08 - 7.72- 56.45 - 8.93 |  54.75 - 1.01 51.74 - 0.48 49.11 - 1.79 | - 53.34 1.94- 50.80 0.87- 48.18 1.64 |
|  **3** | **4** | ***A******B******C*** |  18.73 - 7.22 18.73 - 7.22 18.73 - 7.22 | - 18.69 5.46- 18.69 5.46- 18.69 5.46 |  18.73 - 6.86 17.59 - 7.59 19.86 - 7.67 | - 18.68 5.05- 17.58 5.35- 19.79 5.98 |  19.35 - 33.36 18.01 - 27.41 16.90 - 29.12 | - 19.19 31.85- 17.93 25.69- 16.79 27.46 |
|  **4** | **5** | ***A******B******C*** |  6.26 - 2.81 6.26 - 2.81 6.26 - 2.81 |  - 6.22 - 1.78 - 6.22 - 1.78 - 6.22 - 1.78 |  6.21 - 2.73 5.85 - 2.85 6.65 - 2.87 |  - 6.20 - 2.00 - 5.92 - 1.78 - 6.55 - 1.57 |  6.87 7.55 6.26 5.66 5.85 6.00 |  - 6.66 - 11.94 - 6.20 - 10.37 - 5.82 - 10.64 |

**5. CONCLUDING REMARKS**

This paper proposes an alternative approach for three phase Newton-Raphson load flow calculation, especially in electric power transmission systems. The developed method is based on the nonlinear power-balanced equations, where the derivation of Jacobian matrices and their elements are fully provided. As confirmed by simulation results, the load flow can exploit the impact of unbalanced load on a three phase power system which is the impact on power losses. High levels of load unbalance created greater losses while the same demand is hold at each unbalance scenario.

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